Implementation of the quantum-walk step operator in lateral quantum dots

K. A. van Hoogdalem[1,2] and M. Blaauboer[1]

[1]*Kavli Institute of Nanoscience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands*
[2]*Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland*

We propose a physical implementation of the step operator of the discrete quantum walk for an electron in a one-dimensional chain of quantum dots. The operating principle of the step operator is based on locally enhanced Zeeman splitting and the role of the quantum coin is played by the spin of the electron. We calculate the probability of successful transfer of the electron in the presence of decoherence due to quantum charge fluctuations, modeled as a bosonic bath. We then analyze two mechanisms for creating locally enhanced Zeeman splitting based on, respectively, locally applied electric and magnetic fields and slanting magnetic fields. Our results imply that a success probability of >90% is feasible under realistic experimental conditions.

I. INTRODUCTION

The quantum walk (or quantum random walk) is the quantum-mechanical analog of the classical random walk and describes the random-walk behavior of a quantum particle. The concept "quantum walk" was formally introduced by Aharonov *et al.*[1] in 1993 and suggested earlier by Feynman.[2] The essential difference with the classical random walk lies in the role of the coin: whereas in the classical random walk the coin is a classical object with two possible measurement outcomes ("heads" or "tails"), in the quantum walk the coin is a quantum-mechanical object—typically a two-level system such as a spin-1/2 particle—which can be measured along different bases and hence has a multisided character. As a result, the quantum walk exhibits strikingly different dynamic behavior compared to its classical counterpart due to interference between different possible paths. One example is faster propagation:[1] the root-mean-square distance from the origin $\langle x \rangle_{rms}$ that is covered by a quantum walker grows linearly with the number of steps N (thus corresponding to ballistic propagation) whereas for the classical random walk $\langle x \rangle_{rms} \sim \sqrt{N}$ (corresponding to diffusive propagation). This property has been exploited to design quantum computing algorithms.[3] Both discrete and continuous-time quantum walks have been extensively studied in recent years,[4] including investigations of decoherence[5] and entanglement between quantum walkers.[6]

As far as implementations of quantum walks in actual physical systems are concerned, several proposals have been put forward for a range of optical and atomic systems such as optical cavities,[7] cavity QED systems,[8] trapped atoms and ions,[9] and linear optical elements.[10] On the experimental side, only a few realizations of quantum walks have been achieved: discrete and continuous quantum walks in NMR quantum systems,[11] discrete quantum walks using linear optical elements[12] and, most recently, a continuous quantum walk in an optical waveguide lattice.[13]

For solid-state systems, no realizations of quantum walks exist so far.[14] Investigating the latter is not only interesting from a quantum information point of view but also because the complex nature of decoherence in solid-state nanosystems holds promise of discovering interesting dynamic behavior in these systems.

In this paper we propose an implementation of a quantum-walk step operator in a solid-state quantum system, which consists of a single electron traveling in a one-dimensional chain of quantum dots. The step operator is the basic unit of the quantum walk: it causes the electron to either move to the left or to the right depending on the state of the quantum coin, which in our model is represented by the spin of the electron. In Sec. II we calculate the spin-dependent transfer probability of the electron from one dot to the next, in the presence of different energy-level splittings in neighboring quantum dots due to locally enhanced Zeeman splitting. In Sec. III we take into account the effects of decoherence due to gate-voltage fluctuations modeled as a bosonic bath.[15] In Sec. IV we then propose two physical mechanisms to achieve locally enhanced level splitting in a quantum dot using local electric and magnetic fields and find that under current experimental circumstances successful implementation of the step operator is possible with >90% probability. Conclusions are summarized in Sec. V.

II. MODEL OF THE STEP OPERATOR

Consider a chain of three quantum dots in series, in which the middle dot (M) is occupied by a single electron, see Fig. 1. The energy-level diagram shows the Zeeman splitting $\Delta_z^{(i)}$ of the lowest two levels in a magnetic field and we assume that this splitting is larger in the middle dot by an amount
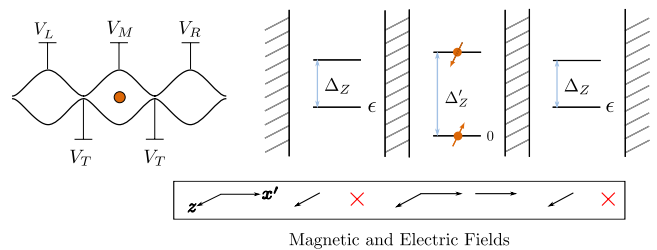

FIG. 1. (Color online) (left) A linear chain of three quantum dots in a magnetic field and (right) the corresponding energy-level diagram. The direction of (locally) applied electric and magnetic fields in each dot is also indicated, a cross indicates no applied field. See the text for further explanation.

$\Delta'_z - \Delta_z$. The gate voltages V_L, V_M, and V_R are used to shift the energy levels in the left, middle, and right dot, respectively, and the gate voltages V_T are used to tune the tunnel coupling between neighboring dots. The initial spin state of the electron is a superposition of spin $\uparrow$ and spin $\downarrow$ and our first goal is to design an implementation of the step operator such that it causes the electron to coherently move to the left (right) if it has spin $\uparrow$ (spin $\downarrow$) and to calculate the probability of successful transfer for this process. To begin with, we assume the right dot to be decoupled from the other two and calculate the spin-dependent probabilities to find the electron in the left and middle dot at a given time. In the absence of decoherence—which is considered further below—the Hamiltonian for the spin-$\uparrow$ and spin-$\downarrow$ components of the electron is given by

$$H_{0,\sigma} = \begin{pmatrix} A_\sigma & \lambda \\ \lambda & B_\sigma \end{pmatrix}, \tag{1}$$

in the basis $\{|L\rangle, |M\rangle\}$. Here $(A_\uparrow, B_\uparrow) = (\epsilon, 0)$, $(A_\downarrow, B_\downarrow) = (\epsilon + \Delta_z, \Delta'_z)$, and λ is the (spin-independent) tunnel coupling between the left and middle dot. The eigenvalues and vectors of Eq. (1) are given by $E_{\pm,\sigma} = \frac{A_\sigma + B_\sigma}{2} \pm \frac{1}{2}\sqrt{(A_\sigma - B_\sigma)^2 + 4\lambda^2}$, $|\psi_{+,\sigma}\rangle = (\sin\theta_\sigma, \cos\theta_\sigma)^T$, and $|\psi_{-,\sigma}\rangle = (\cos\theta_\sigma, -\sin\theta_\sigma)^T$, with $\tan\theta_\sigma \equiv \frac{\hbar\delta_\sigma + \sqrt{(\hbar\delta_\sigma)^2 + 4\lambda^2}}{2\lambda}$, $\delta_\uparrow \equiv \epsilon/\hbar$, and $\delta_\downarrow \equiv (\epsilon + \Delta_z - \Delta'_z)/\hbar$. In the absence of decoherence, the solution of the density-matrix equations $\dot\rho = -(i/\hbar)[H_{0,\sigma}, \rho]$ for the population in the left dot is given by

$$\rho_{LL,\sigma}(t) = \frac{2\lambda^2}{(\hbar\omega_\sigma)^2}[1 - \cos(\omega_\sigma t)] \tag{2}$$

for initial conditions $\rho_{LL,\sigma}(0) = 0$, $\rho_{MM,\sigma}(0) = 1$, and $\hbar\omega_\sigma \equiv \sqrt{(\hbar\delta_\sigma)^2 + 4\lambda^2}$. We see that the probability for the spin-$\uparrow$ component to be in the left dot and the spin-$\downarrow$ component to remain in the middle dot is 1 for $\frac{(\hbar\omega_\uparrow)^2}{2\lambda^2} = 1 - \cos(2n\pi\frac{\omega_\uparrow}{\omega_\downarrow})$, $n = 0, 1, 2, \ldots$. The smallest n that yields a solution to this equation for $\epsilon = 0$ (zero detuning) is $n = 1$, for which $\Delta_z - \Delta'_z = 2\sqrt{3}\lambda$ and $t = 2\pi/\omega_\downarrow$. The second half of the step operator then consists of repeating the same procedure as described above for tunnel coupling between the middle and the right dot, now assuming the left dot to be decoupled.

III. DECOHERENCE DUE TO QUANTUM FLUCTUATIONS

In reality, the time evolution of the occupation probabilities ρ_σ is affected by decoherence due to coupling of the quantum dots to the environment. In typical experimental situations $k_B T < \hbar\omega_\sigma$, for $T \lesssim 100$ mK and $\lambda \lesssim 1.5 \times 10^{-24}$ J (Refs. 16 and 17) so that quantum noise, rather than classical noise, is the dominant source of decoherence. Specifically, since the quantum-walk step operator based on Hamiltonian (1) involves tuning the tunnel couplings λ—resulting in different charge occupations on neighboring dots—the probability distributions will be strongly affected by fluctuations of charge in the environment. The quantum charge fluctuations we consider here are gate-voltage fluctuations, which cause both fluctuations in the tunnel coupling λ and in the energy levels $E_\pm$ of Hamiltonian (1).[18] The

Hamiltonian which describes the quantum-dot system plus the environment is given by

$$H = H_0 + V_\epsilon A_\epsilon + V_\lambda A_\lambda + H_{\text{bath},\epsilon} + H_{\text{bath},\lambda}, \tag{3}$$

with H_0 Hamiltonian (1) of the isolated system and the different terms in the Hamiltonian given by

$$V_\lambda = |L\rangle\langle M| + |M\rangle\langle L|, \tag{4}$$

$$V_\epsilon = |L\rangle\langle L| - |M\rangle\langle M|, \tag{5}$$

$$A_{\epsilon(\lambda)} = \sum_k c_{k,\epsilon(\lambda)}(b^\dagger_{k,\epsilon(\lambda)} + b_{k,\epsilon(\lambda)}), \tag{6}$$

$$H_{\text{bath},\epsilon(\lambda)} = \sum_k \hbar\omega_{k,\epsilon(\lambda)} b^\dagger_{k,\epsilon(\lambda)} b_{k,\epsilon(\lambda)}. \tag{7}$$

We model the environment as a boson bath[19] with creation and annihilation operators $b^\dagger_{k,\epsilon(\lambda)}$ and $b_{k,\epsilon(\lambda)}$ and use the Born-Markov approximation[20] to calculate the time evolution of the spin-dependent occupation probabilities under Hamiltonian (3), assuming weak coupling between the system and the environment and short correlation times of the boson baths (Markovian assumption). The baths are characterized by the symmetric and antisymmetric spectral functions $S^\pm(\omega)$ with $S^+(\omega) = \coth(\frac{\hbar\omega}{2k_B T})S^-(\omega)$ (Ref. 19) and we assume Ohmic baths with Lorentzian damping for which $S^-(\omega) = \alpha\hbar^2\omega\frac{1}{(\omega/\omega_c)^2 + 1}$ with ω_c a cutoff frequency. The time evolution of the reduced density matrix $\rho_{ab}(t)$ is then given by

$$\frac{d\rho_{ab}(t)}{dt} = -i\omega_{ab}\rho_{ab}(t) + \sum_{cd}^{(\text{sec})} R_{abcd}\rho_{cd}(t), \tag{8}$$

with $\omega_{ab} \equiv (E_a - E_b)/\hbar$ and R_{abcd} the Bloch-Redfield tensor.[19,21] Solving this master equation for the two-level system $\{|\psi_{+,\sigma}\rangle, |\psi_{-,\sigma}\rangle\}$ described above for an electron starting at $t = 0$ in the middle dot yields for the evolution of the population ρ_{--} of the ground state $|\psi_-\rangle$ and the coherence terms ρ_{-+},

$$\rho_{--,\sigma}(t) = \frac{1}{2}\left[\tanh\left(\frac{\hbar\omega_\sigma}{2k_B T}\right) - \cos(2\theta_\sigma)\right]e^{-\gamma_1 t} + \frac{\coth\left(\frac{\hbar\omega_\sigma}{2k_B T}\right) - 1}{2\coth\left(\frac{\hbar\omega_\sigma}{2k_B T}\right)}, \tag{9a}$$

$$\rho_{-+,\sigma}(t) = \rho^*_{+-,\sigma}(t) = -\sin\theta_\sigma\cos\theta_\sigma e^{-i\omega_\sigma t}e^{-\gamma_2 t}, \tag{9b}$$

with

$$\gamma_1 = 4\pi^2(\alpha_\lambda\cos^2 2\theta_\sigma + \alpha_\epsilon\sin^2 2\theta_\sigma)\omega_\sigma\coth\left(\frac{\hbar\omega_\sigma}{2k_B T}\right), \tag{10a}$$

$$\gamma_2 = \frac{\gamma_1}{2} + \frac{8\pi^2}{\hbar}(\alpha_\lambda\sin^2 2\theta_\sigma + \alpha_\epsilon\cos^2 2\theta_\sigma)k_B T. \tag{10b}$$

Equations (9) and (10) are valid in the limit γ_1, $\gamma_2 \ll |\hbar\omega_\sigma|$, $1/\bar\tau$ (with $\bar\tau$ the bath correlation time).[20] The survival probability to remain in the middle dot at time t is then given by
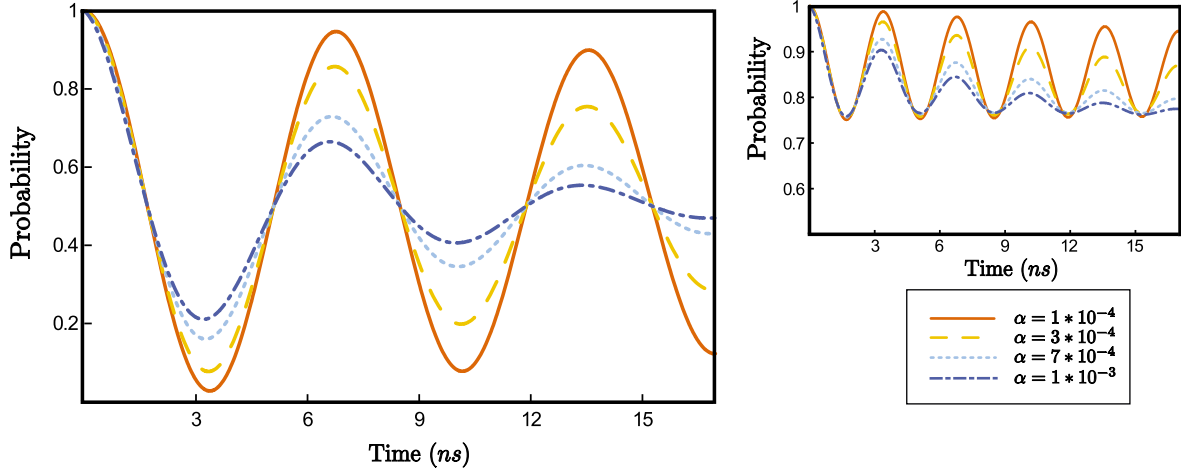
FIG. 2. (Color online) Survival probability [Eq. (11)] of the spin-↑ component (main plot) and spin-↓ component (inset) in the middle dot as a function of time t for various values of $\alpha \equiv \alpha_\lambda = \alpha_\epsilon$ (Ref. 22). Parameters used are $\hbar\delta_\uparrow = 0$, $\hbar\delta_\downarrow = 1$ μeV, $\lambda = (2\sqrt{3})^{-1}$ μeV (Ref. 17), and $T = 10$ mK.

$$P_{M,\sigma}(t) = \frac{1}{2}\left[1 + \cos(2\theta_\sigma)\tanh\left(\frac{\hbar\omega_\sigma}{2k_BT}\right)(1 - e^{-\gamma_1 t}) \right.$$

$$\left. + \cos^2(2\theta_\sigma)e^{-\gamma_1 t} + \sin^2(2\theta_\sigma)\cos(\omega_\sigma t)e^{-\gamma_2 t} \right]$$

$$(11)$$

and plotted for both spin ↑ and spin ↓ in Fig. 2. Using typical experimental parameters (see figure caption and discussion below) we see that successful transfer of the spin-↑ component to the left dot while spin ↓ remains in the middle dot occurs with probability >90% (for $\alpha \leq 3 \times 10^{-4}$) at $t \approx 3.5$ ns.

IV. LOCALLY ENHANCED ZEEMAN SPLITTING

We now consider the question how a different Zeeman splitting in neighboring dots—as assumed in the calculations above—can be realized in practice. In particular, we propose two mechanisms to achieve locally enhanced Zeeman splitting. The first one is by application of a local transverse magnetic field and a local electric field and relies on spin-orbit interaction. Both tunable local magnetic and electric fields have recently been demonstrated experimentally.[23,24] In our model, see Fig. 1, we assume that a local magnetic field (in addition to the global Zeeman-splitting field $\vec{B}$) and a local electric field $\vec{E}(t)$ are applied to the middle quantum dot. In the presence of spin-orbit interaction, the Hamiltonian for this middle dot is then given by

$$H = H_0 + e\vec{E}(t) \cdot \vec{r} + \alpha(p_x\sigma_y - p_y\sigma_x) + \beta(-p_x\sigma_x + p_y\sigma_y),$$

$$(12)$$

with H_0 the Hamiltonian given by Eq. (1), and α and β, respectively, the Rashba and Dresselhaus spin-orbit coupling strengths. Using the Schrieffer-Wolff transformation, the Hamiltonian H can be diagonalized to first order in the spin-orbit terms which yields, for the spin-dependent part,[25]

$$H_{\text{eff}} \equiv e^S H e^{-S} = \frac{1}{2}g^*\mu_B[\vec{B} + \delta\vec{B}(t)] \cdot \vec{\sigma}.$$

$$(13)$$

Here the effective magnetic field $\delta\vec{B}$ is given by $\delta\vec{B} = 2\vec{B} \times [\vec{\Omega}_1(t) + \vec{n} \times \vec{\Omega}_2(t)]$, with $\vec{\Omega}_1(t) = \frac{e\hbar}{E_Z}\alpha_1(E_{y'}/\lambda_-, E_{x'}/\lambda_+, 0)$, $\vec{\Omega}_2(t) = \frac{e\hbar}{E_Z}\beta_1(-E_{x'}/\lambda_-, E_{y'}/\lambda_+, 0)$, $\alpha_1 = \frac{\hbar}{m^*}\frac{E_Z[E_Z^2-(\hbar\omega_0)^2]}{(E_Z^2-E_1^2)(E_Z^2-E_2^2)}$, $\beta_1 = \frac{\hbar}{m^*}\frac{E_Z^2\hbar\omega_c}{(E_Z^2-E_1^2)(E_Z^2-E_2^2)}$, $\lambda_\pm = \hbar/[m^*(\beta \pm \alpha)]$, $E_Z = g^*\mu_B B$, and $E_{1,2} = \hbar(\sqrt{4\omega_0^2 + \omega_c^2} \pm \omega_c)/2$, with $\omega_c = eB/m^*$ the cyclotron frequency and $m^*\omega_0^2 r^2/2$ the harmonic potential of the quantum dot.[26] In these expressions we have used $x' \equiv (x+y)/\sqrt{2}$ and $y' \equiv (y-x)/\sqrt{2}$. Since for the step operator we require the eigenstates of the Hamiltonian H to be $|\uparrow\rangle$ and $|\downarrow\rangle$ we must account for the fact that the eigenvectors of the effective Hamiltonian H_{eff} are transformed by e^S with respect to the eigenvectors of H corresponding to the same eigenvalues. We do this be requiring that the eigenvectors of H_{eff} are $e^S|\uparrow\rangle$ and $e^S|\downarrow\rangle$ to first order in the action S.

Let us assume the global magnetic field $\vec{B}$ to be parallel to the $\hat{z}$ axis. From the expression for H_{eff} it follows that in order to locally generate a different Zeeman splitting (along the $\hat{z}$ axis) in the middle dot compared to its neighboring dots, an additional tunable local magnetic field, e.g., along the $\hat{x}'$ axis, and a tunable local electric field, also along the $\hat{x}'$ axis, are required. We define the total (local) magnetic field $\vec{B}(t) = B_0[n_{x'}(t), 0, n_z]$. We use the expression for the effective magnetic field combined with the requirement for the eigenvectors of H_{eff} to derive two implicit equations for the magnetic and electric field in the $\hat{x}'$ direction as a function of the local magnetic field δB in the $\hat{z}$ direction,

$$n_{x'} - n_z\frac{\gamma\alpha_1 E_{x'}}{\lambda_+} + n_z^2\frac{\gamma\beta_1 E_{x'}}{\lambda_-} = \left(1 + \frac{\delta B}{B_0}\right)\frac{-2c}{1+c^2}, \quad (14)$$
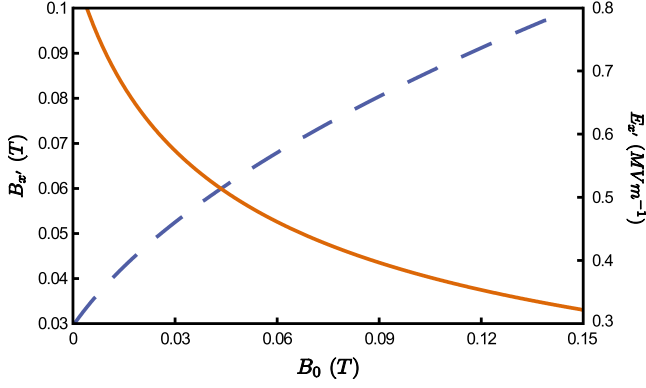
FIG. 3. (Color online) $E_{x'}$ (solid line) and $B_{x'}$ (dashed line) vs B_0 for a local magnetic field δB corresponding to a Zeeman splitting of 1 μeV. Parameters used are $\hbar\omega_0 = 1$ meV (Ref. 26), $m^* = 0.067 m_e$ [for GaAs (Ref. 26)], $\lambda_+ = 2$ μm, $\lambda_- = 10$ μm, (Refs. 23 and 27) and $n_z = 1$.

$$n_z + n_{x'}\frac{\gamma\alpha_1 E_{x'}}{\lambda_+} - n_{x'}n_z\frac{\gamma\beta_1 E_{x'}}{\lambda_-} = \left(1 + \frac{\delta B}{B_0}\right)\frac{1 - c^2}{1 + c^2}, \quad (15)$$

where we have defined $\gamma \equiv 2e\hbar/E_Z$ and $c \equiv \frac{eE_{x'}}{m\omega_0^2}[\frac{1}{\lambda_+} + \frac{n_z\alpha_2}{\lambda_-} - (n_{x'}^2 + n_z^2)\frac{\beta_2'}{\lambda_+}]\sigma_{y'}$.

Figure 3 shows the required fields $B_{x'}$ and $E_{x'}$ as a function of B_0 for typical experimental parameters and a local Zeeman splitting of 1 μeV.[28] We see that in order to have an additional Zeeman splitting of 1 μeV (as assumed in Fig. 2), e.g., at $B_0 = 0.05$ T magnetic and electric fields $B_{x'} \sim 63$ mT and $E_{x'} \sim 4.6 \times 10^5$ Vm^{-1} are required. Although these values of $B_{x'}$ and $E_{x'}$ are (somewhat) larger than those that have been used so far in experiments,[23,29] they may well become available in the near future. Alternatively, one could use larger B_0 (corresponding to larger $B_{x'}$ and smaller $E_{x'}$) or smaller δB_z (corresponding to smaller $B_{x'}$ and $E_{x'}$ but also smaller success probability). In addition, we note that in order to preserve the qubit's quantum state, the fields need to be switched on and off adiabatically. The corresponding switching time T of, e.g., the tunable magnetic field then has to fulfill[30] $T \gg \frac{\hbar n_{x'}}{4n_z|g^*|\mu_B B_0} \sim 0.1$ ns for $n_{x'}/n_z = 0.08$. This time scale is well within experimental reach and compatible with the operation time (duration of electron transfer) ≈ 3.5 ns that we estimated above.

Another method to generate different Zeeman fields in neighboring dots is by using a slanting magnetic field. The latter has recently been demonstrated for the first time by integrating a microsize ferromagnet in a double-dot device.[31] As a result of the slanting field, the orbital and spin degrees of freedom become hybridized, leading to an effective mixed charge-spin two-level system, where the role of the coin is played by the pseudospin instead of the real spin. A global magnetic field ~ 2 T magnetizes the ferromagnet and its inhomogeneity leads to a different Zeeman field in the two quantum dots of $\delta B_z \sim 10$ mT $[|\Delta_z - \Delta_z'| \approx 4 \times 10^{-26}$ J (Ref. 31)]. An advantage of this method compared to using spin-orbit interaction (as discussed above) is that no tunable magnetic fields but only tunable gate voltages are needed. A disadvantage is that the qubit is likely to be more sensitive to orbital decoherence.

Finally, we briefly discuss the probability of successful implementation of a quantum walk along a chain of quantum dots. If p denotes the probability of successful implementation of the first half of the step operator [Eq. (11)], the probability of successful implementation of N consecutive steps can be estimated as $p^{N(N+1)}$. Taking $p = 0.98$, we find that after three steps the probability of success is $> 78\%$ and after ten steps still $> 10\%$, large enough to be statistically visible in repeated runs of an experiment.

V. CONCLUSION

In conclusion, we have proposed an implementation of a discrete quantum-walk step operator in a solid-state nanostructure consisting of a linear chain of three quantum dots. For currently available techniques to locally create enhanced Zeeman splitting and taking into account decoherence due to quantum charge fluctuations, we have analyzed the probability for coherent transfer of the electron to the left or right (conditioned on its spin), and predict that $> 90\%$ success probability is feasible. We hope that our results will stimulate a proof-of-principle experimental demonstration of the discrete quantum-walk step operator in a solid-state nanosystem.

[1] Y. Aharonov, L. Davidovich, and N. Zagury, Phys. Rev. A **48**, 1687 (1993).

[2] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).

[3] V. Kendon, Philos. Trans. R. Soc. London, Ser. A **364**, 3407 (2006).

[4] Two recent reviews on quantum walks are J. Kempe, Contemp. Phys. **44**, 307 (2003); V. Kendon, Math. Struct. Comp. Sci. **17**, 1169 (2007).

[5] T. A. Brun, H. A. Carteret, and A. Ambainis, Phys. Rev. Lett. **91**, 130602 (2003); A. Romanelli, R. Siri, G. Abal, A. Auyuanet, and R. Donangelo, Physica A **347**, 137 (2005); L. Ermann, J. P. Paz, and M. Saraceno, Phys. Rev. A **73**, 012302 (2006); N. V. Prokof'ev and P. C. E. Stamp, *ibid.* **74**, 020102 (2006); J. Košik, V. Bužek, and M. Hillery, *ibid.* **74**, 022310 (2006); A. P. Hines and P. C. E. Stamp, Can. J. Phys. **86**, 541 (2008).

[6] S. E. Venegas-Andraca, J. L. Ball, K. Burnett, and S. Bose, New J. Phys. **7**, 221 (2005); G. Abal, R. Siri, A. Romanelli, and R. Donangelo, Phys. Rev. A **73**, 042302 (2006); Y. Omar, N. Paunković, L. Sheridan and S. Bose, *ibid.* **74**, 042304 (2006).

[7] P. L. Knight, E. Roldan, and J. E. Sipe, J. Opt. Commun. **227**, 147 (2003).

[8] B. C. Sanders, S. D. Bartlett, B. Tregenna, and P. L. Knight, Phys. Rev. A **67**, 042305 (2003); G. S. Agarwal and P. K. Pathak, *ibid.* **72**, 033815 (2005).

[9] B. C. Travaglione and G. J. Milburn, Phys. Rev. A **65**, 032310 (2002); W. Dür, R. Raussendorf, V. M. Kendon, and H.-J. Briegel, *ibid.* **66**, 052319 (2002); K. Eckert, J. Mompart, G. Birkl, and M. Lewenstein, *ibid.* **72**, 012327 (2005); Z.-Y. Ma, K. Burnett, M. B. d'Arcy, and S. A. Gardiner, *ibid.* **73**, 013401 (2006).

[10] P. K. Pathak and G. S. Agarwal, Phys. Rev. A **75**, 032351 (2007).

[11] J. Du, H. Li, X. Xu, M. Shi, J. Wu, X. Zhou, and R. Han, Phys. Rev. A **67**, 042316 (2003); C. A. Ryan, M. Laforest, J. C. Boileau, and R. Laflamme, *ibid.* **72**, 062317 (2005).

[12] B. Do, M. L. Stohler, S. Balasubramanian, D. S. Elliott, C. Eash, E. Fischbach, M. A. Fischbach, A. Mills, and B. Zwickl, J. Opt. Soc. Am. B **22**, 499 (2005); P. Zhang, X.-F. Ren, X.-B. Zou, B. H. Liu, Y.-F. Huang, and G.-C. Guo, Phys. Rev. A **75**, 052310 (2007).

[13] H. B. Perets, Y. Lahini, F. Pozzi, M. Sorel, R. Morandotti, and Y. Silberberg, Phys. Rev. Lett. **100**, 170506 (2008).

[14] A recent proposal for implementation of a quantum algorithm using NAND operations in a tree of quantum dots relies on the continuous time quantum walk, see J. M. Taylor, arXiv:0708.1484 (unpublished).

[15] In Ref. 14 the effects of dephasing are taken into account in a phenomenological way and modeled by a dephasing rate τ_0.

[16] See, e.g., J. R. Petta, A. C. Johnson, J. M. Taylor, E. A. Laird, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Science **309**, 2180 (2005).

[17] F. H. L. Koppens, J. A. Folk, J. M. Elzerman, R. Hanson, L. H. Willems van Beveren, I. T. Vink, H. P. Tranitz, W. Wegschneider, L. P. Kouwenhoven, and L. M. K. Vandersypen, Science **309**, 1346 (2005).

[18] A. Romito and Y. Gefen, Phys. Rev. B **76**, 195318 (2007). In our model fluctuations of the tunnel amplitudes and energy differences, which are both caused by gate voltage and background charge fluctuations, are indeed independent; This assumption is inspired by *ab initio* electronic-structure calculations of double quantum dots with realistic gate profiles [see, e.g., M. Stopa, A. Vidan, T. Hatano, S. Tarucha, and R. M. Westervelt, Physica E (Amsterdam) **34**, 616 (2006), and references therein]; We neglect $1/f$ noise, which becomes important at time scales [estimated as $\tau_{1/f} = \hbar / \sqrt{2\pi\hbar\omega_o k_B T}$] for a sub-Ohmic bath, see A.

Shnirman, Y. Makhlin, and G. Schön, Phys. Scr. **T102**, 147 (2002)] which are larger than the duration of the step operator for the parameters used in Fig. 2; We also neglect fluctuations in the nuclear-spin environment since these are slow [on the order of microseconds or more, see, e.g., J. M. Taylor, J. R. Petta, A. C. Johnson, A. Yacoby, C. M. Marcus, and M. D. Lukin, Phys. Rev. B **76**, 035315 (2007)] compared to the time required to implement the step operator.

[19] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999).

[20] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1998).

[21] The sum extends over terms with $\omega_{ab} - \omega_{cd} \ll 1/\Delta t$, (secular constraint) where Δt is the time scale of the Markovian course-grained evolution (Ref. 20).

[22] Values of α in this range were measured by O. Astafiev, Yu. A. Pashkin, Y. Nakamura, T. Yamamoto, and J. S. Tsai, Phys. Rev. Lett. **93**, 267007 (2004).

[23] K. C. Nowack, F. H. L. Koppens, Yu. V. Nazarov, and L. M. K. Vandersypen, Science **318**, 1430 (2007).

[24] F. H. L. Koppens, C. Buizert, K. J. Tielrooij, I. T. Vink, K. C. Nowack, T. Meunier, L. P. Kouwenhoven, and L. M. K. Vandersypen, Nature (London) **442**, 766 (2006).

[25] M. Borhani, V. N. Golovach, and D. Loss, Phys. Rev. B **73**, 155311 (2006); V. N. Golovach, M. Borhani, and D. Loss, *ibid.* **74**, 165319 (2006).

[26] For a recent review on spin qubits in quantum dots see R. Hanson, L. P. Kouwenhoven, J. R. Petta, S. Tarucha, and L. M. K. Vandersypen, Rev. Mod. Phys. **79**, 1217 (2007).

[27] J. B. Miller, D. M. Zumbühl, C. M. Marcus, and Y. B. Lyanda-Geller, D. Goldhaber-Gordon, K. Campman, and A. C. Gossard, Phys. Rev. Lett. **90**, 076807 (2003); S. Amasha, K. MacLean, I. P. Radu, D. M. Zumbühl, M. A. Kastner, M. P. Hanson, and A. C. Gossard, *ibid.* **100**, 046803 (2008).

[28] Similar values for $B_{x'}$ and $E_{x'}$ are found for an in-plane global magnetic field and out-of-plane local fields.

[29] E. A. Laird, C. Barthel, E. I. Rashba, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Phys. Rev. Lett. **99**, 246601 (2007).

[30] A. Messiah, *Quantum Mechanics* (Dover, New York, 1999), Chap. XVII.

[31] M. Pioro-Ladrière, T. Obata, Y. Tokura, Y.-S. Shin, T. Kubo, K. Yoshida, T. Taniyama, and S. Tarucha, Nat. Phys. **4**, 776 (2008); Y. Tokura, W. G. van der Wiel, T. Obata, and S. Tarucha, Phys. Rev. Lett. **96**, 047202 (2006).